

# Deformed Hořava-Lifshitz Cosmology and Stability of Einstein Static Universe

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## Abstract

Stability of the Einstein static universe versus the linear scalar, vector and tensor perturbations is investigated in the context of deformed Hořava-Lifshitz cosmology inspired by entropic force scenario. A general stability condition against the linear scalar perturbations is obtained. Using this general condition, it is shown that there is no stable Einstein static universe for the case of flat universe,  $k = 0$ . For the the special case of large values of running parameter of HL gravity  $\omega$ , in a positively curved universe  $k > 0$ , the domination of the quintessence and phantom matter fields with barotropic equation of state parameter  $\beta < -\frac{1}{3}$  is necessary while for a negatively curved universe  $k < 0$ , the matter fields with  $\beta > -\frac{1}{3}$  are needed to be the dominant fields of the universe. Also, a neutral stability against the vector perturbations is obtained. Finally, an inequality including the cosmological parameters of the Einstein static universe is obtained for the stability against the tensor perturbations. It turns out that for large  $\omega$  values, there is a stability against the tensor perturbations.

Keywords: Hořava-Lifshitz cosmology, Einstein static universe, stability

## 1 Introduction

While near a century has passed since the birth of General Relativity (GR), we still do not have a clear understanding of the origin of the gravitational force. In the quest for discovering the nature of this mysterious force, we are faced with two significant problems. The first is the lack of a theoretical framework to reconcile two prosperous theories; the quantum mechanics and GR at the Planck scale. The second is the absence of a unified theory of gravitational force with other three fundamental forces. Contrary to the common approaches for solving these problems, E. Verlinde [1] by continuing the works done by Sakharov [2], Jacobson [3] and Padmanabhan [4], claimed that the gravity is not a fundamental but is an emerging force in the spacetime. Indeed, gravity can be described as an entropic force due to the changes of information on the holographic screen when a test particle situated at an arbitrary distance from the screen is shifting toward it. Generally, the approach of Verlinde is built on two main pillars; the holographic principle and the equipartition rule which reproduces Newton's laws and Einstein's field equations. Undoubtedly, if the validity of the current interpretation of gravity be approved, then it will affect the direction of research in theoretical physics at the future. As an example, the author of [5] by using the Verlinde's hypothesis, extracted the Newtonian gravity in loop quantum gravity. It is also interesting that in [6], the entropic force is considered as the origin of the coulomb force. In [7], it is shown that the holographic dark energy will be derivable from the

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entropic force interpretation of gravity. The modified Friedmann equations by this notion of gravity are obtained in different setups such as Einstein gravity [8], braneworld scenarios [9], Gauss-Bonnet gravity [10], and Hořava-Lifshitz gravity [11]. There are also other applications of Verlinde's hypothesis, some of which are mentioned in [12].

As mentioned above, theoretical physics suffers from the lack of a quantum theory of gravity. As one of the biggest hurdle in achieving this goal, one can point to the fact that GR is not renormalizable theory at high energy limits (ultra-violet limit (UV)), so there is no control on the theory and its predictions. As one of the efforts made to overcome this big problem, one can refer to a new class of UV complete theory of gravity known as Horava-Lifshitz (HL) gravity [13]. We should note that HL theory of gravity is achieved at the cost of loosing the Lorentz symmetry through a Lifshitz-type anisotropic scaling at high energy limits i.e.  $t \rightarrow t^z$ ,  $x^i \rightarrow l x^i$  where  $z \geq 1$  is the dynamical critical exponent. Therefore, it is a non-relativistic renormalizable theory of gravity. While HL gravity at high energies is non-relativistic, it is expected that the four dimensional general covariance can be recovered at the low energy limits. More technically, in contrast to the standard GR, the HL theory of gravity is not full diffeomorphism invariance, rather it only has a local Galilean invariance [14]. Then, it is expected that HL gravity approaches to GR at the infra-red (IR) limits. However, the HL theory of gravity can be considered as a UV complete candidate for GR. The existence of an anisotropic scaling at the UV limit results in a mechanism for generation of cosmological perturbations so that it can solve the horizon problem without resorting to inflation. It is also a remarkable point that in HL model of gravity, due to the lack of local Hamiltonian constraint, dark matter may appear as an integration constant [15]. In order to a comprehensive review of HL gravity and some of its cosmological implications, see [16]. The total action of HL gravity can be written as [13]

$$S_{HL} = \int dt dx^3 \sqrt{g} N \left[ \frac{2}{K^2} (K_{ij} K^{ij} - \lambda K^2) - \frac{\kappa^2}{\omega^4} C_{ij} C^{ij} + \frac{\kappa^2 \mu}{2\omega^2} \epsilon^{ijk} {}^3R_{il} \nabla_j {}^3R_k^l \right. \\ \left. - \frac{\kappa^2 \mu^2}{8} {}^3R_{il} {}^3R^{ij} + \frac{\kappa^2 \mu^2}{8(3\lambda - 1)} \left( \frac{4\lambda - 1}{4} ({}^3R)^2 - \Lambda {}^3R + 3\Lambda^2 \right) R \right], \quad (1)$$

where  $\mu, \Lambda$  and  $\kappa, \lambda, \omega$  represent the dimensional and dimensionless constant parameters of HL gravity, respectively. Also, the quantities  $K_{ij} = \frac{1}{2N} (\dot{g} - \nabla_i N_j - \nabla_j N_i)$  and  $C^{ij} = \epsilon^{ijk} \nabla_k ({}^3R_l^j - \frac{1}{4} {}^3R \delta_j^i)$  denote the extrinsic curvature and the Cotton tensor, respectively. Note that for the case of  $\lambda \rightarrow 1$ , the kinetic section of action (1) approaches to GR action in IR limit. It should be stressed that in UV limit, the behavior of HL model of gravity is very different from GR. So, as will be discussed in the following, for  $\lambda \rightarrow 1$  HL gravity does not perfectly reproduces full four dimensional diffeomorphism invariance at large distances (IR limit).

One of the most interesting issues in the context of any model of gravity is the black hole solutions. For the first time, the authors of [17] provided a static spherically symmetric black hole solution for asymptotically Lifshitz spacetimes in the presence of running coupling constant  $\lambda$ . Through the study of thermodynamical properties of this solution, it is shown in [18] that for the case of  $\lambda = 1$ , this solution reduces to Reissner-Nordstrom black hole instead of the standard Schwarzschild black hole. More precisely, since the horizon radius of the black hole solution, corresponding to  $\lambda = 1$ , includes a geometric parameter as  $\alpha = \frac{1}{2\omega}$  which can play the role of electric charge, it looks like the Reissner-Nordstrom black hole. While the geometric parameter  $\alpha$  results in the modified entropy of the black hole, for increasing  $\omega$  it becomes smaller and in the limit  $\omega \rightarrow \infty$  it approaches to zero and one recovers the standard Schwarzschild entropy expression,  $S = \frac{A}{4}$ . With these properties, the original action in equation (1) represents the *deformed* HL model of gravity in which one recovers the standard GR in IR limit for case  $\lambda = 1$ , while its black hole solution does not result in reproduction of the usual Schwarzschild black hole solution. Of course, by adding an IR modification term  $\mu^4 ({}^3R)$  to the original HL action (1) this issue will be resolved (see [19] for more details).

In this work, we investigate the Einstein static universe and its stability versus homogeneous scalar, vector and tensor perturbations in the framework of the deformed HL cosmology inspired by entropic force scenario. The main motivation of studying the stability of Einstein static universe comes from the emergent universe scenario [20]. The emergent universe scenario is a past-eternal inflationary model in which the horizon problem is solved before the beginning of inflation and the big-bang singularity is removed. In the framework of this cosmological model, the universe is originated from an Einstein static state rather than a big bang singularity. However, this model suffers from a fine-tuning problem which can be ameliorated by modifying the cosmological field equations of the general relativity. For this reason, analogous static

solutions and their stabilities have been studied in the context of the modified theories of gravity such as  $f(R)$  [23],  $f(T)$  [24], Einstein-Cartan theory [25], massive gravity [26], Lyra geometry [27], loop quantum cosmology [28] and braneworld scenarios [29]. Stability of the Einstein static universe is also studied in the Horava-Lifshitz gravity [30]. Our present paper is based on the model proposed in [11] in which the dynamical equations of the deformed HL gravity for a Friedmann-Robertson-Walker (FRW) background is obtained based on the thermodynamical properties of the deformed HL black holes [18]. In the present framework of deformed HL cosmology inspired by entropic gravity, the modified Friedmann equations and the corresponding results are different from those obtained in Refs. [30]. Throughout this work, we use the units of  $\hbar = c = G = k_B = 1$ .

## 2 Friedmann-Robertson-Walker Universe in Deformed HL Gravity from Entropic Force

In this section, assuming the emergence of gravity in space-time as an entropic force, we plan to derive the modified Friedmann equations in HL cosmological setup. We consider the homogeneous and isotropic FRW background space-time with the metric

$$ds^2 = h_{ab}dx^a dx^b + \tilde{r}^2 d\Omega_2^2, \quad (2)$$

where  $h_{ab} = \text{diag} \left( -1, \frac{a^2}{1-kr^2} \right)$  and  $d\Omega_2^2 = d\theta^2 + \sin^2 d\phi^2$  denote the two dimensional metrics,  $\tilde{r} = a(t)r$  in which  $a(t)$  is the cosmic scale factor and  $k = -1, 0$  or  $1$  corresponds to an open, flat or closed universe, respectively. In a FRW spacetime, one can show that

$$\tilde{r}_A = \frac{1}{\sqrt{H^2 + \frac{k}{a^2}}}, \quad (3)$$

where  $\tilde{r}_A$  and  $H = \frac{\dot{a}}{a}$ , are the dynamical apparent horizon and the Hubble parameter, respectively. In order to have a comprehensive review on trap surfaces and dynamical apparent horizons, see [21]. The temperature on the apparent horizon reads as  $T = \frac{|K|}{2\pi}$  where  $K$  is the surface gravity whose value at the apparent horizon of the FRW universe is given by

$$K = -\frac{1}{\tilde{r}_A} \left( 1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A} \right). \quad (4)$$

By assuming that the universe as a closed system is in the thermodynamical equilibrium, one may ignore the second term in equation (4) and find the following temperature of the apparent horizon as

$$T_A = \frac{1}{2\pi\tilde{r}_A}. \quad (5)$$

In Verlinde's scenario of gravity, the entropy of black hole has a very important role in the derivation of Newton's law of gravitation and Friedmann equation, such that in the presence of any correction, entropy-area relation in Einstein gravity is extended as

$$S = \frac{A}{4} + s(A), \quad (6)$$

where  $A$  and  $s(A)$  are the area of horizon and entropy correction term, respectively. It should be noted that the Loop quantum gravity (LQG) imposes a quantum correction to the horizon area law of a black hole, namely the logarithmic correction [22]. Then, the entropy of black holes in the context of HL gravity in the presence of logarithmic correction, can be written as [18]

$$S = \frac{A}{4} + \frac{\pi}{\omega} \ln(A), \quad (7)$$

where  $\omega$  is the dimensionless running coupling constant of HL gravity, which is proportional to the inverse of dimensionless geometric parameter  $\alpha$  arising from LQG correction [22]. Equation (7) indicates that for

the special case of  $\omega \rightarrow \infty$ , one will recover the Einstein entropy expression. In this process, by using the continuity equation

$$\dot{\rho} + 3H(\rho + p) = 0, \quad (8)$$

and the energy equipartition rule

$$E = \frac{1}{2}NT, \quad (9)$$

the Raychaudhuri equation can be expressed as

$$\frac{1}{2}NdT + \frac{1}{2}TdT = 4\pi\tilde{r}_A^3(\rho + p)Hdt, \quad (10)$$

where  $N$  denotes the number of bits on the screen and is proportional to the area of the screen (horizon), because of  $N = 4S$ . As has already been mentioned, the approach of Verlinde is strongly influenced by the holographic principle and equipartition rule. Therefore, inspired by holographic principle and assuming that the screen has a total energy  $E$ , which is distributed between all the bits  $N$ , the temperature of the screen  $T$  is given by the equipartition rule in equation (9). Integrating Raychaudhuri equation (10) will give the following modified Friedmann equation in the deformed HL cosmological setup inspired by the entropic force [11]

$$\frac{1}{2\omega}(H^2 + \frac{k}{a^2})^2 + H^2 + \frac{k}{a^2} = \frac{8\pi}{3}\rho. \quad (11)$$

As we see, the Friedmann equation of the standard model of cosmology is modified by the presence of the first term. As the simplest case, i.e. for the flat universe  $k = 0$ , the Friedmann equation is modified by a  $\frac{H^4}{2\omega}$  term. Also, by differentiation of equation (11) with respect to the cosmic time and using the continuity equation (8) the following modified acceleration equation can be obtained

$$\frac{\ddot{a}}{a} = \omega \left[ -1 + (1 + \frac{16\pi}{3\omega}\rho)^{\frac{1}{2}} \right] - 4\pi \left[ 1 + \frac{16\pi}{3\omega}\rho \right]^{-\frac{1}{2}}(\rho + p). \quad (12)$$

It is easy to check that equations (11) and (12) in IR limit,  $\omega \rightarrow \infty$ , will reduce to the following standard Friedmann equations as

$$H^2 + \frac{k}{a^2} = \frac{8\pi}{3}\rho. \quad (13)$$

and

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}(\rho + 3p). \quad (14)$$

### 3 Einstein Static Universe, Scalar Perturbations and Stability Analysis

In what follows, we will consider the barotropic equation of state  $p(t) = \beta\rho(t)$  and will expand the acceleration equation (12), keeping up to the third order of the energy density  $\rho(t)$ . The Einstein static universe in the deformed HL cosmological setup inspired by entropic gravity can be obtained by the condition  $\ddot{a} = \dot{a} = 0$ , through the equations (11) and (12) as

$$\frac{k^2}{2\omega a_0^4} + \frac{k}{a_0^2} = \frac{8\pi}{3}\rho_0, \quad (15)$$

and

$$\omega \left[ \frac{8\pi}{3\omega}\rho_0 - \frac{1}{8}(\frac{16\pi}{3\omega}\rho_0)^2 + \frac{1}{16}(\frac{16\pi}{3\omega}\rho_0)^3 + \dots \right] - 4\pi \left[ 1 - \frac{8\pi}{3\omega}\rho_0 + \frac{3}{8}(\frac{16\pi}{3\omega}\rho_0)^2 + \dots \right] \rho_0(1 + \beta) = 0, \quad (16)$$

where  $a_0$  and  $\rho_0$  refer to the scale factor and the energy density of the Einstein static universe, respectively. We consider the linear homogeneous scalar perturbations around the Einstein static universe, given

in equations (15) and (16), and investigate their stability versus these perturbations. The perturbations in the cosmic scale factor  $a(t)$  and the energy density  $\rho(t)$  depending only on time can be represented by

$$\begin{aligned} a(t) &\rightarrow a_0(1 + \delta a(t)), \\ \rho(t) &\rightarrow \rho_0(1 + \delta \rho(t)). \end{aligned} \quad (17)$$

Substituting (17) into equation (11) and using equation (15), via linearizing the perturbation terms, leads to the following equation

$$-\left(\frac{k^2}{\omega a_0^4} + \frac{k}{a_0^2}\right)\delta a = \frac{4\pi}{3}\rho_0\delta\rho. \quad (18)$$

By applying the same method on equations (12) and (16), we get

$$\ddot{\delta a} = \frac{4\pi}{3}\rho_0\delta\rho \left[ -1 - 3\beta + \frac{16\pi}{3\omega}(2 + 3\beta)\rho_0 - \frac{32\pi^2}{3\omega^2}(7 + 9\beta)\rho_0^2 \right], \quad (19)$$

where substituting equation (18) into (19) gives the following equation

$$\ddot{\delta a} + \left(\frac{k^2}{\omega a_0^4} + \frac{k}{a_0^2}\right) \left[ -1 - 3\beta + \frac{16\pi}{3\omega}(2 + 3\beta)\rho_0 - \frac{32\pi^2}{3\omega^2}(7 + 9\beta)\rho_0^2 \right] \delta a = 0. \quad (20)$$

Then, in order to have the oscillating perturbation modes representing the existence of a stable Einstein static universe in the framework of the deformed HL cosmology from entropic gravity, the following condition should be satisfied

$$\left(\frac{k^2}{\omega a_0^4} + \frac{k}{a_0^2}\right) \left[ -1 - 3\beta + \frac{16\pi}{3\omega}(2 + 3\beta)\rho_0 - \frac{32\pi^2}{3\omega^2}(7 + 9\beta)\rho_0^2 \right] > 0, \quad (21)$$

which leads to the following solution for the equation (20)

$$\delta a = C_1 e^{iAt} + C_2 e^{-iAt}, \quad (22)$$

where  $C_1$  and  $C_2$  are integration constants and  $A$  is given by

$$A = \left(\frac{k^2}{\omega a_0^4} + \frac{k}{a_0^2}\right)^{\frac{1}{2}} \left[ -1 - 3\beta + \frac{16\pi}{3\omega}(2 + 3\beta)\rho_0 - \frac{32\pi^2}{3\omega^2}(7 + 9\beta)\rho_0^2 \right]^{\frac{1}{2}}. \quad (23)$$

The stability condition, together with the inequality (21), gives us the following different class of solutions:

- For the case of the flat universe,  $k = 0$ , there is no stable Einstein static universe in the framework of the deformed HL cosmological setup inspired by entropic gravity.
- For large  $\omega$  values for which the first term in the parenthesis as well as the third and fourth terms in the brackets in equation (21) vanish, we will have the following condition

$$\frac{k}{a_0^2}(-1 - 3\beta) > 0, \quad (24)$$

which leads to the following cases:

1. The case of  $k > 0$  with the equation of state parameter  $\beta < -\frac{1}{3}$ , representing the quintessence and phantom fields.
2. The case of  $k < 0$  with the equation of state parameter  $\beta > -\frac{1}{3}$ , representing ordinary matter fields.

- For a non-flat universe with an arbitrary  $\omega$  values, the general constraint in equation (21) must be satisfied which shows an interplay between the cosmological parameters  $a_0$ ,  $\rho_0$ ,  $k$ ,  $\omega$  and  $\beta$  of this model.

## 4 Vector and Tensor Perturbations and Stability Analysis

In this section, we study the stability analysis of the Einstein static universe against the vector and tensor perturbations. In a cosmological context, the vector perturbations of a perfect fluid having energy density  $\rho(t)$  with a barotropic pressure  $p(t) = \beta\rho(t)$  are governed by the co-moving dimensionless *vorticity* defined as  $\varpi_a = a\varpi$  whose modes satisfy the following propagation equation

$$\dot{\varpi}_\kappa + (1 - 3c_s^2)H\varpi_\kappa = 0, \quad (25)$$

where  $c_s^2 = dp/d\rho$  is the sound speed and  $H$  is the Hubble parameter [31]. This equation is valid in our treatment of Einstein static universe in the framework of the deformed HL gravity derived from entropic force scenario through the modified Friedmann equations (11) and (12). For the Einstein static universe with  $H = 0$ , equation (25) reduces to

$$\dot{\varpi}_\kappa = 0. \quad (26)$$

Equation (26) represents that the initial vector perturbations will remain frozen. Then, independent of the values of the the cosmological parameters  $a_0$ ,  $\rho_0$ ,  $k$ ,  $\omega$  and  $\beta$  of this model, there is a neutral stability against vector perturbations.

The tensor perturbations, namely gravitational-wave perturbations, of a perfect fluid are described by the co-moving dimensionless transverse-traceless shear  $\Sigma_{ab} = a\sigma_{ab}$ , whose modes satisfy the following equation

$$\ddot{\Sigma}_\kappa + 3H\dot{\Sigma}_\kappa + \left[ \frac{\mathcal{K}^2}{a^2} + \frac{2k}{a^2} - \frac{8\pi}{3}(1 + 3\omega)\rho \right] \Sigma_\kappa = 0, \quad (27)$$

where  $\mathcal{K}$  is the co-moving index  $D^2 \rightarrow -\mathcal{K}^2/a^2$  in which  $D^2$  is the covariant spatial Laplacian [31]. One can show that for the Einstein static universe, this equation by using equations (11), (12) and (15), takes the form

$$\ddot{\Sigma}_\kappa + \left[ \frac{1}{a_0^2}(\mathcal{K}^2 - \frac{k^2}{\omega a_0^2}) - 2\omega \left( 1 - (1 + \frac{16\pi}{3\omega}\rho_0)^{\frac{1}{2}} \right) \right] \Sigma_\kappa = 0. \quad (28)$$

Then, in order to have the stable modes against the tensor perturbations, the following inequality should be satisfied

$$\frac{1}{a_0^2}(\mathcal{K}^2 - \frac{k^2}{\omega a_0^2}) - 2\omega \left( 1 - (1 + \frac{16\pi}{3\omega}\rho_0)^{\frac{1}{2}} \right) > 0. \quad (29)$$

Using the expansion  $(1 + x)^{1/2} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + O(x^3)$ <sup>1</sup>, we find the following inequality for very large  $\omega$  values

$$\frac{1}{a_0^2}\mathcal{K}^2 + \frac{16\pi}{3}\rho_0 > 0, \quad (30)$$

which is always satisfied because of  $\rho_0 > 0$ . Therefore, for very large  $\omega$  values, there is a stability against the tensor perturbations in the framework of Horava-Lifshitz gravity inspired by entropic gravity. For an arbitrary and not so large  $\omega$  values, the inequality (29) indicates an interplay between the cosmological parameters  $a_0$ ,  $\omega$ ,  $\mathcal{K}$ ,  $k$  and  $\rho_0$  of the Einstein static universe.

## 5 Conclusion

The existence of the big bang singularity in the early universe is one of the essential problems in standard cosmology. To solve this problem in the framework of GR, the so called "emergent universe scenario" [20] was introduced as an inflationary cosmology without the initial singularity. Based on this scenario, the early universe before the transition to the inflationary phase, has an initial state, known as the Einstein static state. In the framework of this cosmological model, the universe has emerged from an Einstein static state rather than a big bang singularity. Clearly, variety of the perturbations in the early universe can affect the stability of the initial static state. On the other hand, the classical GR is not a appropriate theory at high energy states so that early universe is highly influenced by various physical conditions which may result in some modifications of GR. By this motivation, we have investigated the stability of initial Einstein static

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<sup>1</sup>Note that we have used the above expansion relation also in obtaining the equations (16) and (19).

state against the linear homogeneous scalar, vector and tensor perturbations within the framework of the new class of UV complete theory of gravity known as the deformed HL gravity inspired by entropic force scenario. It is shown that there is no stable Einstein static universe against the linear scalar perturbations, for the case of flat universe,  $k = 0$ . In the case of large values of dynamical parameter of HL gravity  $\omega$ , for a positively curved universe  $k > 0$ , the domination of the quintessence and phantom matter fields with the barotropic equation of state parameter  $\beta < -\frac{1}{3}$  is necessary while for a negatively curved universe  $k < 0$ , the matter fields with  $\beta > -\frac{1}{3}$  are needed to be the dominant fields of the universe. There is a neutral stability against the vector perturbations. For the tensor perturbations, an inequality including the parameters  $a_0$ ,  $\omega$ ,  $\mathcal{K}$ ,  $k$  and  $\rho_0$  of the Einstein static universe is obtained which certainly accounts for a stability, for large  $\omega$  values.

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